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Numerical study of two classes of cellular automaton models for traffic flow on a two-lane roadway

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Abstract. In this paper, we present computer simulation results of traffic flow on a two-lane roadway with different types of vehicles, cars and trucks for example. We consider two classes of two-lane traffic cellular automaton models, namely the well known Nagel-Schreckenberg model and an extension of the Fukui-Ishibashi model. These two models, which differ in their acceleration limits, show an important differences in their fundamental diagrams, lane-changing and ping-pong behaviors. Moreover, we investigate the importance of braking noise and the proportion of trucks on the traffic flow of a two-lane roadway.

PACS. 89.40.+k Transportation – 05.70.Fh Phase transitions: general studies – 05.65.+b Self-organized systems – 05.40.Jc Brownian motion

1 Introduction

In the last years, vehicular traffic problems have attracted much attention, and a number of cellular automaton (CA) models describing the traffic flow have been proposed [1–4]. For a realistic description of traffic on highways the idealized single-lane models were generalized to develop CA models of two-lane traffic [5–8]. Several attempts have been made so far in this direction, and different lane-changing procedures have been proposed [9–13]. These procedures can be symmetric or asymmetric with respect to the lanes. In the same way, if there are different types of vehicles with different maximum possible speeds noted by $v_{\max}$ (cars and trucks for example), the lane-changing rules can be symmetric or asymmetric with respect to the vehicles.

In two-lane traffic it is of particular interest to investigate systems with different types of vehicles. Chowdhury *et al.* [14] considered a two-lane ring with slow and fast vehicles (different $v_{\max}$) evolving in both lanes. The simulation results have shown that already for small densities, the fast vehicles take on average the free-flow velocity of the slow vehicles, even if only a small fraction of slow vehicles have been considered. Recent simulation results of Knospe *et al.* [12] show that the influence of slow vehicles seems to be overestimated by the multi-lane variants of the Nagel-Schreckenberg model (NaSch model). In order to weaken the effect of slow vehicles they considered anticipation effects, *i.e.* the driver estimates the velocity of the vehicle in the next time step. They show that anticipation reduces the influence of slow vehicles drastically.

In this paper, we want to go one step beyond that and look for systematic slowing of cars caused by the trucks. Thus, we propose here a 2-D extended version of the 1-D Fukui-Ishibashi model, elaborated by Wang *et al.* [15], for single lane traffic to take into account the exchange of vehicles between the first and second lanes. The Wang *et al.* model, denoted hereafter by "WWH model", is a CA model where the velocities of vehicles are determined by Fukui-Ishibashi rules with stochastic delay for vehicles following the trail of the vehicle ahead, *i.e.* no driver would like to slow down when far away from the vehicles ahead. In the high density case, the stochastic delay in this model represents better safety (assurance of the avoidance of crashes) than that of the Fukui-Ishibashi model, and leads to much higher asymptotic average velocities of traffic flow than NaSch model. As the found in [15], the diagram of traffic flow against density, which we call hereafter the "fundamental diagram", is quite different from the NaSch and Fukui-Ishibashi models even in the simplest case of $v_{\max} = 1$. Throughout this work, the two-lane traffic WWH model is compared to the corresponding NaSch model [12,13].

We introduce two kinds of vehicles on the circuit. Here, we will denote by "cars" the fast vehicles and "trucks" the slow ones. We study the effect of trucks on the two-lane traffic flow for different traffic situations. The lane-changing rules are those elaborated by Knospe *et al.* [12] and can be symmetric or asymmetric with respect to lanes or to the vehicles. We limit this work to the symmetric case with respect to the lanes but the rules governing lane changes can be symmetric or not with respect to vehicles. Hence, for the asymmetric situation, the first lane is considered *fast* and the second is *slow*: only cars can exchange

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between the two lanes (symmetric rule with respect to the lanes) while the trucks always move on the same (*slow*) lane, *i.e.* lane change is forbidden. For the symmetric situation, both cars and trucks use the two lanes.

In order to investigate the role of the forward movement of vehicles in two-lane traffic, we shall use two different classes of models, namely the very well known NaSch model [3] and the recently proposed model of Wang *et al.* [15]. For the lane changing rules we choose those proposed by Knospe *et al.* [12] in order to account for the effects of interactions between the vehicles in the two lanes. Our model is a CA defined on the two 1-D lattices of $2*N$ sites with periodic boundary conditions, forming a closed circuit. The outline of this paper is as follows: In the next section the forward movement and the lane changing rules are defined. Simulation results and their discussion is presented in Sections 3 and 4 where we give a detailed analysis of the influence of trucks on the two-lane traffic. Finally, we summarize our findings and present some concluding remarks in Section 5.

2 Forward movement and lane changing rules

The NaSch model is a CA model which is described as follows: On a ring with L sites every site can either be empty or occupied by one vehicle with velocity $v = 0, 1, ..., v_{\max}$. Let *gap* be the number of empty sites in front of the car, and v its velocity at time t. At each discrete time step the arrangement of N cars is updated in parallel according to the following rules:

1) Acceleration: with regard to the vehicle ahead $v' \leftarrow \min(v + 1, gap, v_{\max})$;
2) Noise: with a probability p: $v'' \leftarrow \max(v' - 1, 0)$;
3) Movement: the car moves v'' sites ahead.

The WWH model is a cellular automaton traffic flow model between the Fukui-Ishibashi [4] and NaSch models. The WWH model adopts the acceleration of the Fukui-Ishibashi model and the NaSch stochastic delay only for cars following the trail of the car ahead. Hence, WWH adopt the following rules:

1) Acceleration: with regard to the vehicle ahead $v' \leftarrow \min(gap, v_{\max})$;
2) Noise: with a probability p: $v'' \leftarrow \max(v' - 1, 0)$ if $gap \le v_{\max}$;
3) Movement: the car moves v'' sites ahead.

The single lane model is not capable of modelling realistic traffic for several reasons, primarily because realistic traffic is usually composed of vehicles with different limiting velocities evolving in multi-lane traffic. In this work, we consider a two-lane model consisting of two single lanes with periodic boundary conditions where additional rules defining the exchange of vehicles between the lanes are introduced. For the NaSch forward movement, we restrict ourselves to the exchange rules of Knospe *et al.* [12] which are defined by the following two criteria (see Fig. 1):

• Incentive criterion:
1. $v_{hope} > gap$, with $v_{hope} = \min(v + 1, v_{\max})$;

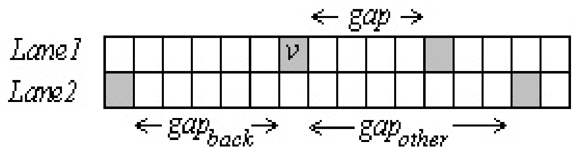

Fig. 1. Illustration of the quantities relevant for the lane changing rules in the two-lane system. The hatched cells are occupied by vehicles. Interest is focused on the vehicle noted by v.

• Safety criteria:
2. $gap_{other} > gap$;
3. $gap_{back} \ge v_{\max}$.

On the one hand, if we assume that the safety criteria hold and we adopt the Knospe lane-changing procedure to the WWH model, a vehicle with $gap > v + 1$ will not change lanes and its velocity should be equal to $\min(gap, v_{\max})$. However, if we suppose that this vehicle changes lanes then its velocity will be equal to $\min(gap_{other}, v_{\max})$ which is higher than $\min(gap, v_{\max})$ according to the safety criteria. This leads to an optimization of the vehicle velocity. Thus, this shows that the Knospe procedure of lane-changing is incompatible with the WWH forward movement. On the other hand, since the acceleration step of the WWH forward movement is given by $v_{t+1} = \min(gap, v_{\max})$ then it is obvious to adopt the following WWH lane-changing procedure:

• Incentive criterion:
1. $v_{hope} > gap$, with $v_{hope} = v_{\max}$;
• Safety criteria:
2. $gap_{other} > gap$;
3. $gap_{back} \ge v_{\max}$.

In general, the update in the two-lane models is divided into two sub-steps: in one sub-step, the vehicles may change lanes in parallel following the lane changing rules and in the other sub-step each vehicle may move effectively by the forward movement rules as in the single-lane traffic. On the other hand, two situations are examined for lane changing rules: the symmetric case and the asymmetric case with respect to the vehicles. In the former case the lane-changing criteria are applied equally to both cars and trucks. In the latter, the lane-changing rules are applied only to cars.

In order to present the simulation results for the two models explained above, we define as ρ the total density of vehicles on the two lanes. In the initial state, the vehicles are randomly distributed at sites of the system and the percentage of trucks is taken to be 10%. We adopt the temporal treatment of Nagatani [10], *i.e.* on odd time steps, in the first lane, each vehicle moves forward or changes to the second lane in parallel according to lane-changing rules. On even time steps, the second lane is considered in the same manner as the first one. Moreover, we consider two different situations: the homogeneous case where the two-lane system is occupied by fast vehicles only and the inhomogeneous case where both types of vehicles evolve in the system. The results are obtained from numerical simulations on a lattice of 2×10^3 sites with a hundred random

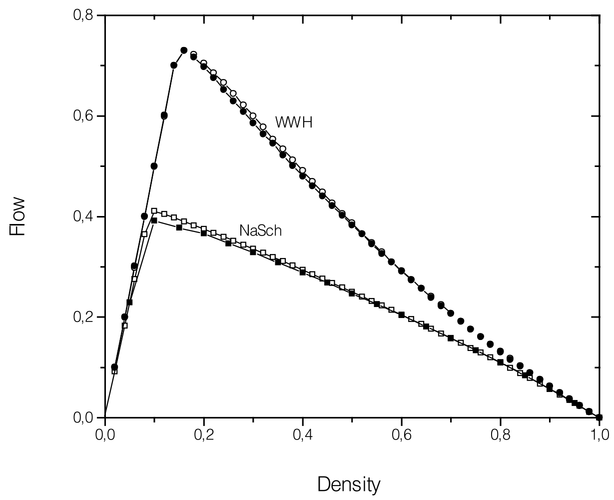

Fig. 2. Fundamental diagrams of individual lanes in the two-lane models (open symbols) for homogeneous systems compared with the corresponding single-lane diagrams for NaSch and WWH models (solid symbols), $v_{\max} = 5$ and $p = 0.4$.

initial configurations of vehicles. For each initial configuration, results are obtained by averaging over 5×10^4 time steps after the first 1×10^4 time-steps, so that the system reaches a stationary state.

3 Simulations of traffic without trucks in the circuit

We examine first of all the behavior of fast vehicles in the homogeneous case (no slow cars in the circuit). The lane changing rules are applied equally in the two lanes (symmetric case with respect to the lanes). These rules sets are relevant for the traffic in towns and on highways, where overtaking in both lanes is allowed. The fundamental diagram of the individual lanes in both models used is shown in Figure 2, where we have included the curves corresponding to the case of a single-lane road for braking noise of $p = 0.4$. We note first that the average flow per lane in both lanes are quite similar and that the simulations reproduce well-known results, *e.g.* an increase of the maximum flow per lane compared to the flow of a single-lane model. In spite of additional disturbances introduced in traffic *via* the lane changing rules, the general effects are beneficial. However, this effect is very weak in the WWH model as shown in Figure 2. In the same way, the behavior of flow shows a maximum at low densities for both models used, for the NaSch model, the flow reaches a maximum at $\rho_{j\,\max} \approx 0.1$, which is at or near a sharp bend of the flow curve, while this maximum is reached at appreciably higher densities $\rho_{j\,\max} \approx 0.16$ in the WWH model.

To get further insight into the lane changing dynamics, another interesting quantity to look at is the frequency of lane changes at different densities. Hence, Figures 3 and 4 show the variations of the lane-changing frequency per car against the total density ρ of cars for the two proposed models. For the NaSch model, the maximum number of

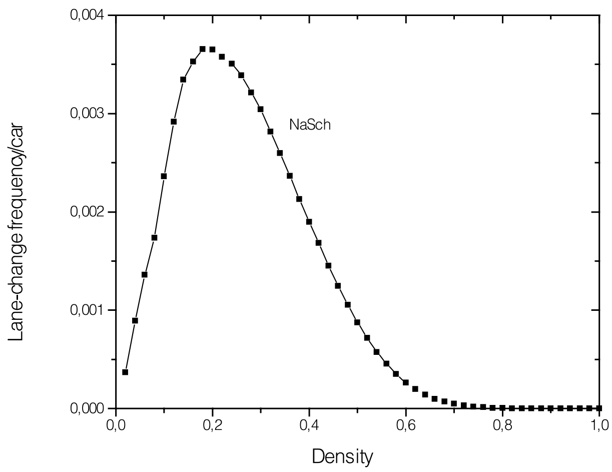

Fig. 3. Density dependence of frequency lane-changes per car in the NaSch model for a homogeneous system with $v_{\max} = 5$ and $p = 0.4$.

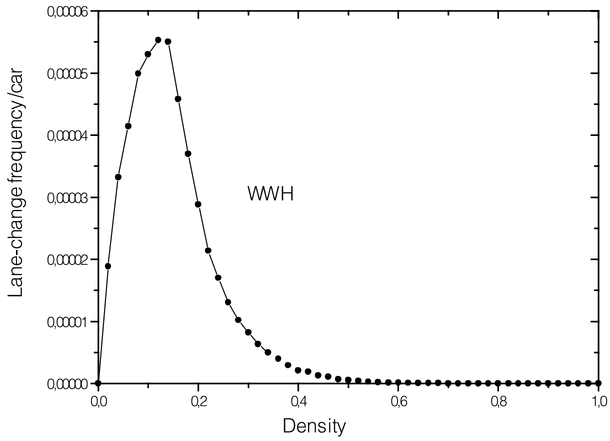

Fig. 4. Density dependence of frequency lane-changes per car in the WWH model for a homogeneous system with $v_{\max} = 5$ and $p = 0.4$.

lane changes occurs at densities much higher than $\rho_{j\,\max}$ whereas it is reached near $\rho_{j\,\max}$ in the WWH model. This is due to the fact that for the latter model, the velocities of vehicles are solely determined by the distances to the vehicles ahead. Moreover, at small densities, the incentive and safety criteria are almost never fulfilled since the mean gap between consecutive vehicles is always greater than $v_{\max}$. With increasing density this gap becomes increasingly smaller than $v_{\max}$ which encourages the drivers to change lanes. Comparing the two classes of models, the frequency of lane-changes is too much higher in the NaSch model than that of WWH. This is due primarily to the low acceleration in the NaSch model (equal to 1), which leads to fast formation of plugs compared to the WWH model which belongs to the class of large acceleration traffic models.

An artifact of the described models is easily recognizable when one starts with all cars on the same lane, say the first one. Assuming fairly high density, then all cars see somebody in front of them, but nobody on the second

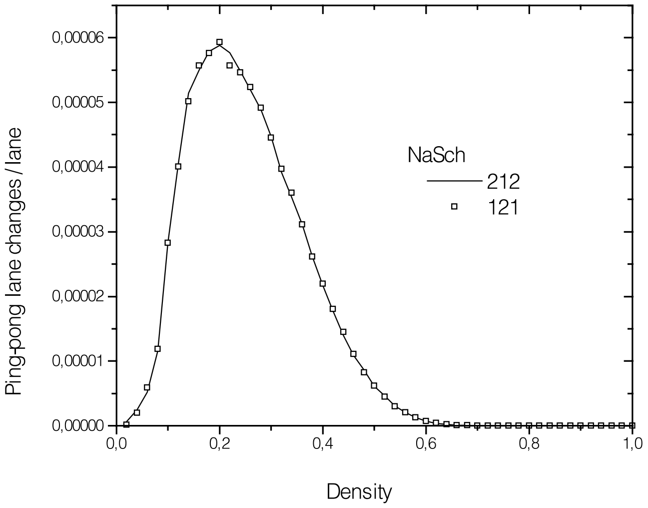

Fig. 5. Density dependence of ping-pong lane-changes per lane in the NaSch model for $p = 0.4$. The variations of *121*-ping-pong (indicated by symbols) are fairly similar to *212*-ping-pong (indicated by solid curves).

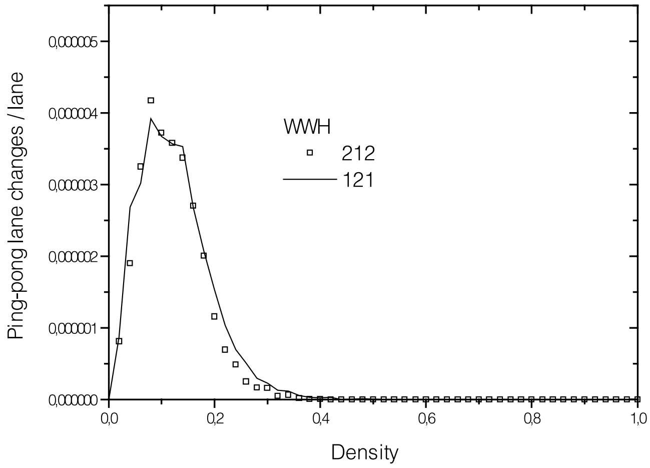

Fig. 6. Density dependence of ping-pong lane-changes per lane in the WWH model for $p = 0.4$. The variations of *121*-ping-pong (indicated by solid curve) are fairly similar to *212*-ping-pong (indicated by symbols).

lane. In consequence, everybody decides to change to the second lane, so that all cars end up on the second lane. Here, they now all decide to change to the first lane again, etc., such that these coordinated lane changes go on for a long time (*ping-pong effect).* To overcome this difficulty, one randomizes the lane changing decision but the flow-density curves are only marginally changed [11].

The frequency of "ping-pong lane changes" can be determined as follows: a car makes two lane changes in two consecutive iterations. Obviously, there are first-second-first (*121*) and second-first-second (*212*) ping-pong lane changes. Figures 5 and 6 show the evolution frequency of (*121*) and (*212*) ping-pong lanes changes for both NaSch and WWH models respectively. It is clear that these curves actually look fairly similar (due to the symmetry of the lane changing rules) and that they decrease in general with high densities. Yet, for the WWH model the number of ping-pong lane changes is negligible compared with the NaSch one and this offers better correspondence with real traffic where the phenomenon of ping-pong is rare or essentially non-existent. The behavior of the ping-pong lane changes frequency is naturally the same as the lane-change frequency in the two classes of models. Moreover, at small densities as found in [11], about 50% of the lane changes are ping-pong changes.

4 Simulations of traffic with trucks in the circuit

We now consider the inhomogeneous case which is obviously more relevant for practical purposes. We consider two different situations: a) the symmetric case where the fast as well as the slow vehicles may use both lanes, *e.g.* both categories of vehicles are treated equally with respect to the lane changing rules and b) the asymmetric case where the trucks are introduced only in the second lane, namely the trucks are constrained to move forward in the second lane and are not allowed to change to the first lane, whereas the fast vehicles may use both lanes. In this case the lane change rules are applied solely to the cars. Hence, the lane 1 and 2 are considered as *fast* and *slow* lanes respectively.

It is clear that the introduction of the slow vehicles to the system, even for small proportions, introduces a considerable disruption of traffic. Indeed, two trucks driving side by side on different lanes can form a plug which blocks the succeeding traffic and then leads to the formation of a platoon. This induces drastic traffic flow reduction [9,12]. To reduce the formation of plugs, several attempts have been done for the NaSch traffic model such as anticipation and sequential updates which lead to slightly increased values for the maximum flow [12]. Another way of minimizing these effects is to start with all of the trucks in the second lane only, which are then not permitted to change lane [13].

Fundamental diagrams

To show the effects of slow vehicles on two-lane systems for the two classes of models, we consider the inhomogeneous flows for the symmetric and asymmetric versions. Hence, initial conditions with 10% trucks with $v_{\max}^{trucks} = 3$ and 90% cars with $v_{\max}^{cars} = 5$ with braking noises equal for both types of vehicles ($p_{cars} = p_{trucks} = 0.4$) analogous to references [9,12] were selected for the symmetric version, it is shown that the flows per lane in the first and the second lane are fairly similar, which is due to the symmetry of the lane changing update rules. In contrast, in the case of asymmetric vehicle distribution, the flow in the second lane is dominated by trucks, while the flow in the first lane is higher than the corresponding symmetric model (Figs. 7 and 8). For the two classes of models, at low densities, cars can overtake trucks more effectively and then

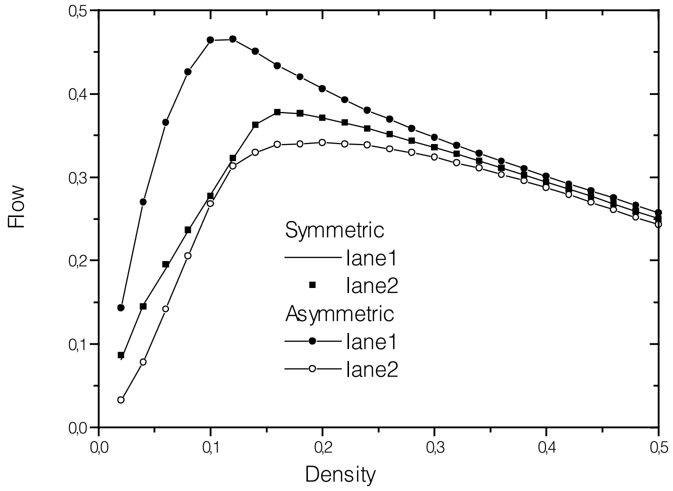

Fig. 7. Comparison of the flow per lane of a two-lane system with 10% trucks, ($p_{cars} = p_{trucks} = 0.4$ and $v_{\max}^{cars} = 5$, $v_{\max}^{trucks} = 3$) in the NaSch model for the symmetric and the asymmetric versions of the lane changing rules.

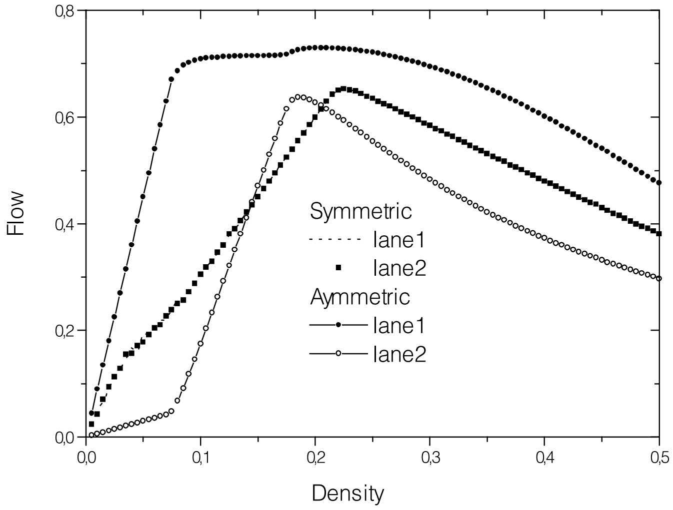

Fig. 8. Comparison of the flow per lane of a two-lane system with 10% trucks, ($p_{cars} = p_{trucks} = 0.4$ and $v_{\max}^{cars} = 5$, $v_{\max}^{trucks} = 3$) in the WWH model for the symmetric and the asymmetric versions of the lane changing rules.

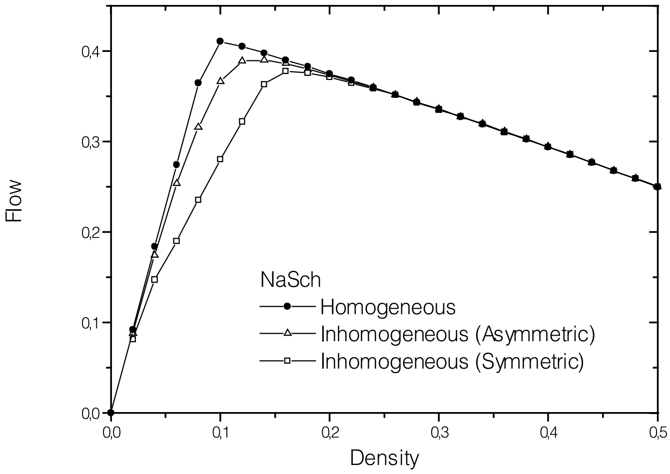

Fig. 9. Comparison of homogeneous with inhomogeneous total flows (asymmetric and symmetric cases) for the NaSch model, $p_{cars} = p_{trucks} = 0.4$.

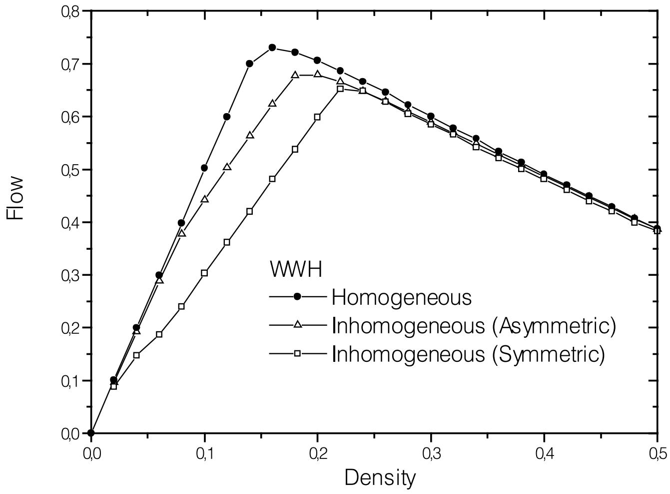

Fig. 10. Comparison of homogeneous with inhomogeneous total flows (asymmetric and symmetric cases) for the WWH model, $p_{cars} = p_{trucks} = 0.4$.

the slow lane is dominated by trucks in the asymmetric version, while the flow in the fast lane is slightly greater than the flow of the homogeneous model. In the two classes of models, for relatively low densities, we found that the homogeneous total flow is greater than its corresponding asymmetric inhomogeneous case which is in turn greater than that of the symmetric inhomogeneous case (Figs. 9 and 10). Moreover, unlike the NaSch model, the plugs are partially avoided in the WWH model and we obtain generally higher values for the average flows.

Now, it was assumed that the slow vehicles have smaller braking parameters than the fast ones, and simulations for the two classes of traffic models for $p_{cars} = 0.4$, $p_{trucks} = 0.125$ and 10% of trucks were conducted. For the NaSch model, it is shown that in both symmetric and asymmetric versions, the inhomogeneous total flow exceeds the capacity of a homogeneous system at a certain density of vehicles (Fig. 11); this phenomenon cannot appear in realistic traffic. This paradoxical effect found in the NaSch model can be explained as follows, at small densities the evolution of cars is not obstructed by trucks. As soon as the density becomes relatively significant, the cars take on average the velocity of trucks *i.e.* the total flow is dominated by that of the trucks. This behaves in a manner similar to real traffic with vehicles with $v_{\max} = 3$ and with $p = 0.125$. Moreover, we verified that at relatively high densities, the homogeneous traffic flow with parameters ($v_{\max} = 5,\ p = 0.4$) becomes smaller than that corresponding to ($v_{\max} = 3,\ p = 0.125$). Consequently, at relatively high densities, the inhomogeneous total flow will be greater than that of an homogeneous system. In contrast, the WWH model does not seem to be significantly affected by differences in the braking probability of

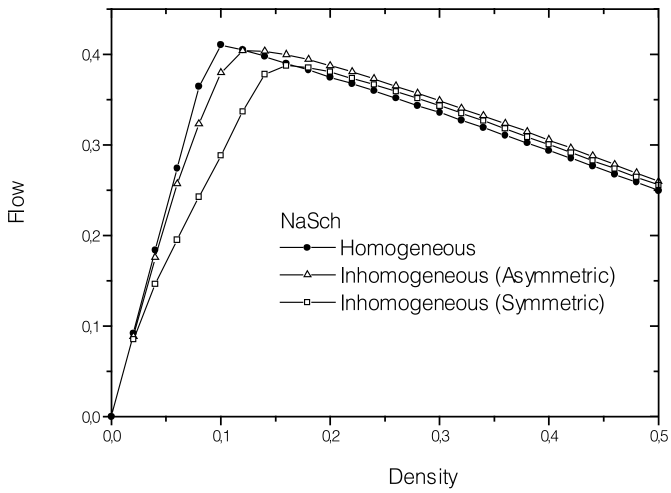

Fig. 11. Comparison of homogeneous with inhomogeneous total flows (asymmetric and symmetric cases) for the NaSch model, $p_{cars} = 0.4$ and $p_{trucks} = 0.125$.

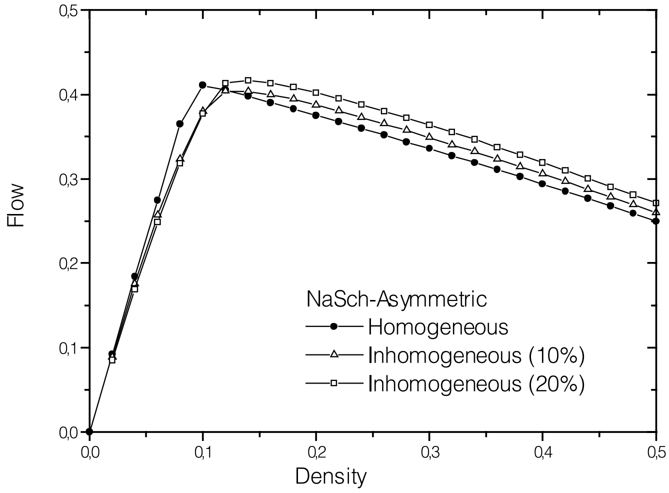

Fig. 13. Comparison of homogeneous with inhomogeneous total flows (10% and 20% of trucks) in the asymmetric version of the NaSch model, $p_{cars} = 0.4$ and $p_{trucks} = 0.125$.

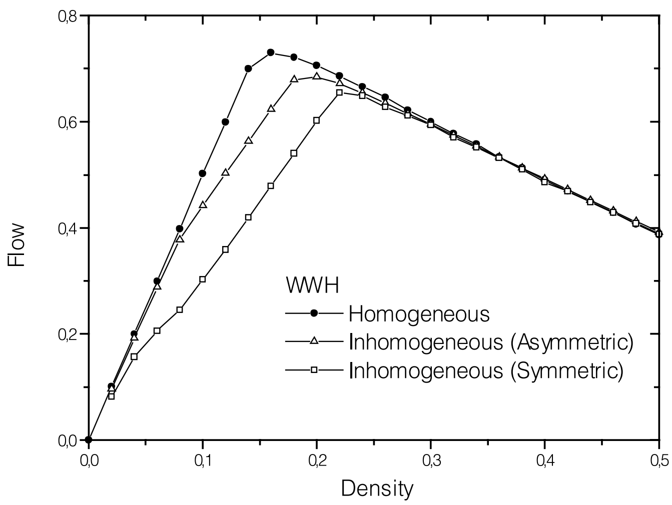

Fig. 12. Comparison of homogeneous with inhomogeneous total flows (asymmetric and symmetric cases) for the WWH model, $p_{cars} = 0.4$ and $p_{trucks} = 0.125$.

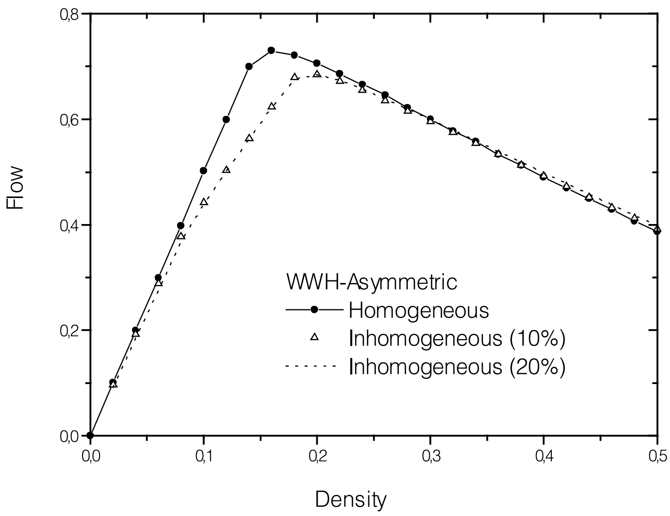

Fig. 14. Comparison of homogeneous with inhomogeneous total flows (10% and 20% of trucks) in the asymmetric version of WWH model, $p_{cars} = 0.4$ and $p_{trucks} = 0.125$.

vehicles (Fig. 12). Now for the two classes of models, it is easy to see that if we assign a smaller braking probability to the fast vehicles ($p_{cars} = 0.125$ and $p_{trucks} = 0.4$), the flow of the inhomogeneous system is always smaller than that corresponding to the homogeneous system.

In order to quantify the effect of the proportion of slow vehicles and its effects on the phenomenon described above, simulations with initial conditions of 10% and 20% trucks were run for ($p_{cars} = 0.4$, $p_{trucks} = 0.125$). Hence, the paradoxical effect observed in the NaSch model increases with the percentage of slow vehicles for both versions of symmetry (see for example the asymmetric case in Fig. 13). However the WWH model exhibits only small changes in the fundamental diagrams when we increase the proportion of trucks in the circuit (Fig. 14).

Frequency and ping-pong lane-changes

For the two classes of models, we investigated the behavior of the frequency and ping-pong lane-changes for the symmetric and asymmetric versions. The percentage of trucks in the circuit is 10% and the braking noises are taken as equal for both types of vehicles ($p_{cars} = p_{trucks} = 0.4$). The results of our simulations for the density dependence of the lane changing frequency are given in Figures 15 and 16 where a comparison of this quantity in both symmetric and asymmetric situations is shown. We note first that for the NaSch model at low densities the lane changing frequency for the symmetric version exceeds that corresponding to the asymmetric version whereas for high densities the reverse is the case. Moreover, the maximum of the lane changing frequency in the two examined versions is reached at the same value of the density ($\rho \approx 0.1$).

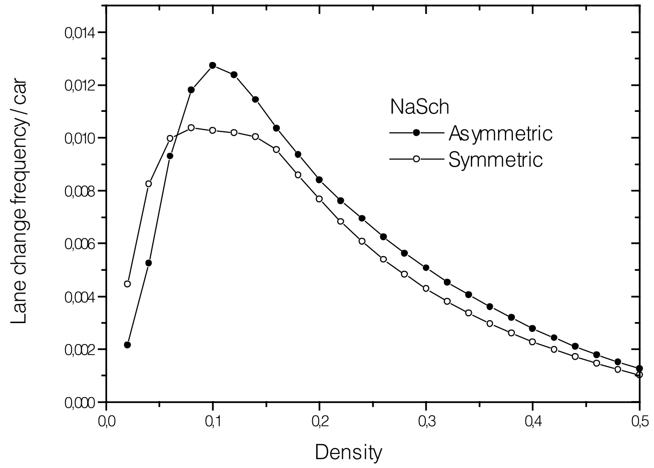

Fig. 15. Density dependence of frequency lane-changes per car in the NaSch model for the symmetric (open symbols) and the asymmetric (solid symbols) cases for $p_{cars} = p_{trucks} = 0.4$ and $v_{\max}^{cars} = 5$, $v_{\max}^{trucks} = 3$.

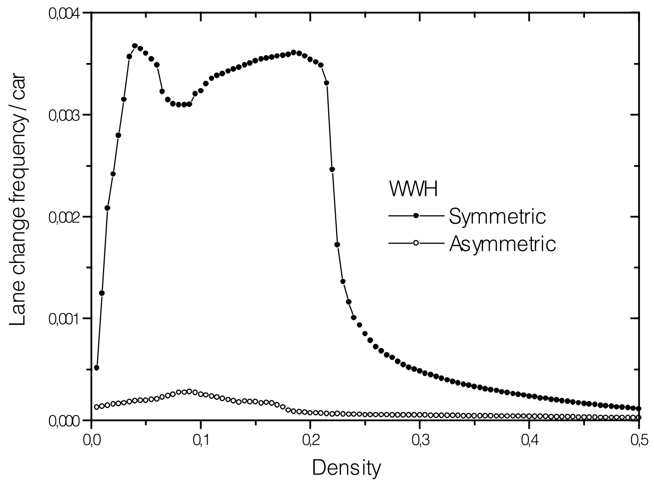

Fig. 16. Density dependence of frequency lane-changes per car in the WWH model for the symmetric (solid symbols) and the asymmetric (open symbols) cases for $p_{cars} = p_{trucks} = 0.4$ and $v_{\max}^{cars} = 5$, $v_{\max}^{trucks} = 3$.

In contrast, for the WWH model, the number of lane changes in the symmetric version is always superior than the corresponding asymmetric version. Comparing the two classes of models, we notice that the WWH model allows very few lane changes than that of NaSch one and especially in the asymmetric version. As a consequence of this behavior, the plugs in the WWH traffic model are very few compared to the corresponding NaSch model.

Next, we shall study the behavior of ping-pong lane changes in the inhomogeneous system (Figs. 17 and 18). For the two classes of traffic models and in the same way as in the homogeneous case, the identical evolutions of (*121*) and (*212*) ping-pong lane changes in the symmetric version reflect the symmetry lane changing rules. Moreover, the asymmetry of lane changing rules (fast lane-slow lane) leads to (*212*) ping-pong preference and induces a decrease of the individual flow of the slow lane (lane 2) compared to

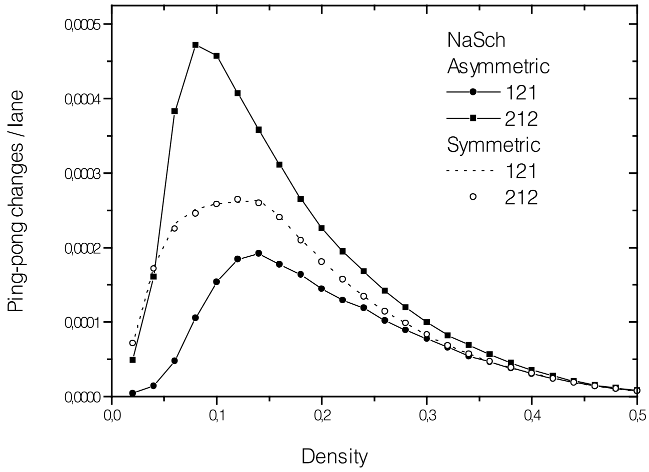

Fig. 17. Density dependence of ping-pong lane-changes per lane in the NaSch model: in asymmetric and symmetric cases for $p_{cars} = p_{trucks} = 0.4$ and $v_{\max}^{cars} = 5$, $v_{\max}^{trucks} = 3$.

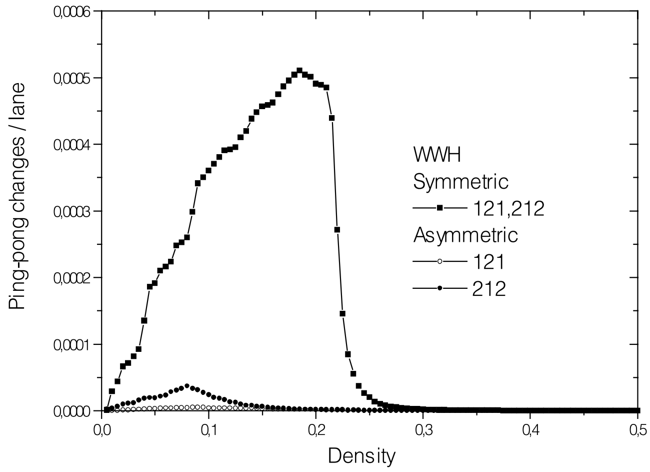

Fig. 18. Density dependence of ping-pong lane-changes per lane in the WWH model: in asymmetric and symmetric cases for $p_{cars} = p_{trucks} = 0.4$ and $v_{\max}^{cars} = 5$, $v_{\max}^{trucks} = 3$.

that of the fast lane (lane 1) (see Figs. 7 and 8). Finally, we point out that the introduction of trucks increases drastically the lane change frequency as well as the number of ping-pong lane changes. This induces a decrease in the inhomogeneous total flow compared to that of the homogeneous case (see Figs. 9 and 10).

5 Conclusion

In summary, we have presented a comparison of stochastic cellular automaton models for the traffic flow in a two-lane roadway with two species of vehicles and for two versions of symmetry. For the two classes of models we have shown that the introduction of trucks in the circuit induces a considerable disruption of highway traffic. Hence, our findings confirm the well-known result that the slow cars in two-lane systems dominate the behavior of the whole system even for a small number of slow vehicles. Moreover, in both classes of models, the asymmetric total flow always exceeds the corresponding symmetric one. On the other hand, in the NaSch model, the inhomogeneous total flow exceeds the capacity of a homogeneous system from a certain density of vehicles if we assign a small braking noise p to slow vehicles compared to that of the fast ones ($p_{trucks} < p_{cars}$). However, this paradoxical effect observed in the NaSch model doesn't appear in the WWH model. Finally, by comparing the ping-pong frequencies, we found that WWH model allows very few ping-pong lane changes compared to the NaSch case (especially in the asymmetric version). As a consequence of this behavior, the two-lane traffic is found to be more fluid in the WWH model compared to the NaSch one *i.e.* for the two versions of symmetry, the WWH-flow exceeds the NaSch-flow at all densities.

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